6.4 Warmup

Prove the following identities. State any restrictions on the variables.

1. $$\sec x\left(\sec x - \cos x\right) = \tan^2 x$$

2. $$\frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} = \tan x$$

6.4 Solving Trigonometric Equations Using Identities

Solving some trigonometric equations may require making substitutions using trigonometric identities to try and write the equation so that there is only one trigonometric function being used.

Solve the following equations algebraically over the specified domain.

1. $\cos x + \sin 2x = 0$ $\quad 0 \le x < 2\pi$

$\cos x + 2\sin x\cos x = 0$

$\cos x(1 + 2\sin x) = 0$

$\cos x = 0$ $\quad 1 + 2\sin x = 0$

$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ $\quad \sin x = -\frac{1}{2}$ $\quad x_r = 30° \text{ or } \frac{\pi}{6}$

Q3: $x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

Q4: $x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$x = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

① use an identity to try and rewrite with a single function sin, cos or tan.

② factor if necessary when one side = 0

③ use unit circle or reference angles

2. $2\cos x - 2 = \sin^2 x$ $\quad 0 \le x < 2\pi$

$2\cos x - 2 = 1 - \cos^2 x$

$-1 \quad -1$

$+\cos^2 x \quad +\cos^2 x$

$\cos^2 x + 2\cos x - 3 = 0$

$(\cos x - 1)(\cos x + 3) = 0$

$\cos x = 1$ $\quad \cos x = -3$ → no solution

$x = 0, \cancel{2\pi}$

$x = 0$

3. $\sin^2 x = \frac{\sqrt{3}}{2}\tan x \cos x \qquad 0 \le x < 360^0$

$\sin^2 x = \frac{\sqrt{3}}{2}\frac{\sin x}{\cos x}\cdot \cos x$

$\sin^2 x - \frac{\sqrt{3}}{2}\sin x = 0$

$\sin x\left(\sin x - \frac{\sqrt{3}}{2}\right) = 0$

$\sin x = 0 \qquad \sin x = \frac{\sqrt{3}}{2} \qquad x_R = 60^\circ$

$x = 0^\circ, 180^\circ$

Q1 : $x = 60^\circ$

Q2 : $x = 180^\circ - 60^\circ$

$x = \{0^\circ, 120^\circ, 60^\circ, 180^\circ\}$

4. $\cos 2x = -\cos x$ Solve over the reals

$2\cos^2 x - 1 = -\cos x$

5. $\sin x = \sqrt{3}\cos x$ Solve over the reals

$\frac{\sin x}{\cos x} = \frac{\sqrt{3}\cos x}{\cos x}$

$\frac{\sin x}{\cos x} = \sqrt{3}$

$\tan x = \sqrt{3}$

$\sec x = \frac{1}{\cos x}$

6. $3\cos x - 2 = 5\sec x$ Solve over the reals p320 # 1-5, 7-12, 15

$\cos x \cdot 3\cos x - 2 = 5 \times \frac{1}{\cos x} \times \cos x$

$3\cos^2 x - 2\cos x = 5$

$3\cos^2 x - 2\cos x - 5 = 0$