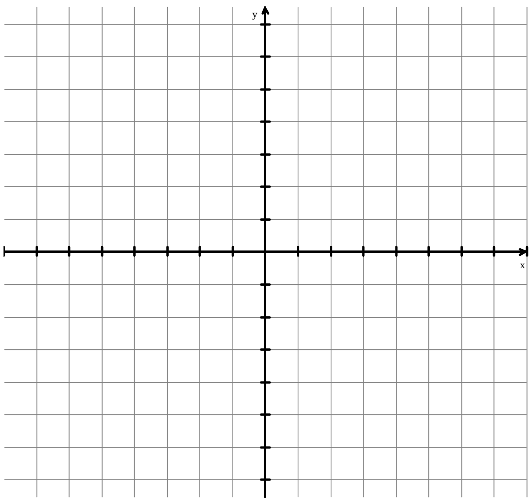
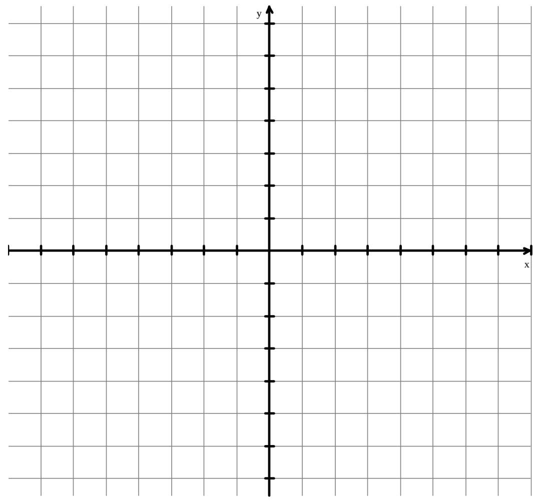
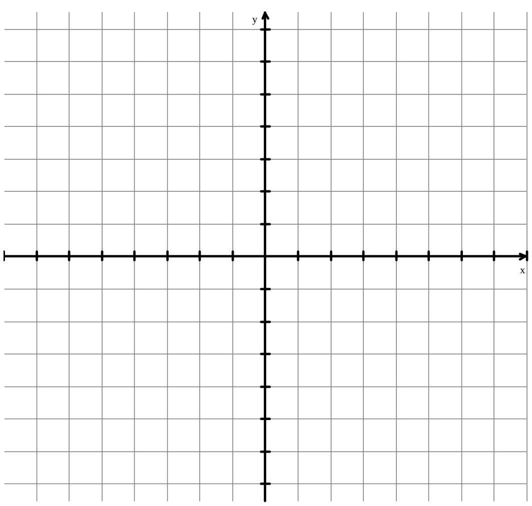
PC12 2.2 Warm-Up

Graph each of the following. State the domain and range.

1. $y = \sqrt{x+2}$ 	2. $y + 2 = \sqrt{x-5}$ 
3. $-y = \sqrt{\frac{1}{2}x + 3}$ 	

2.2 Square Root of a Function

The function $y = \sqrt{f(x)}$ is the square root of the function $y = f(x)$ for the values of $f(x) \geq 0$. We can see the difference between the two functions by looking at how we would calculate the y values for the following related functions:

- For $y = 2x - 3$, we would multiply x by 2 and subtract 3 to arrive at y.
- For $y = \sqrt{2x-3}$, we would multiply x by 2, subtract 3, then take the square root to arrive at y.

The only difference between the two sets of operations is the final step of taking the square root in the second equation. If we look at a table of values for the two functions, we can see that the y values for $y = \sqrt{2x-3}$ are simply the square roots of the y values for $y = 2x - 3$.

x	$y = 2x - 3$	$y = \sqrt{2x-3}$
0	-3	np.
2	1	1
6	9	3
14	25	5
26	49	7

cannot $\sqrt{\text{negative}}$

Using the information above, determine the relative locations for $y = \sqrt{f(x)}$ in each given interval of $y = f(x)$.

Value of $f(x)$	$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
Relative location of graph of $y = \sqrt{f(x)}$	notpossible	$\sqrt{f(x)} = 0$ invariant point	$\sqrt{f(x)} > f(x)$	$\sqrt{f(x)} = 1$ invariant point	$\sqrt{f(x)} < f(x)$

Remember: Invariant points are points that are shared between $y = f(x)$ and $y = \sqrt{f(x)}$. In this case, the invariant points occur when $f(x) =$ ____0____ and when $f(x) =$ ____1____.

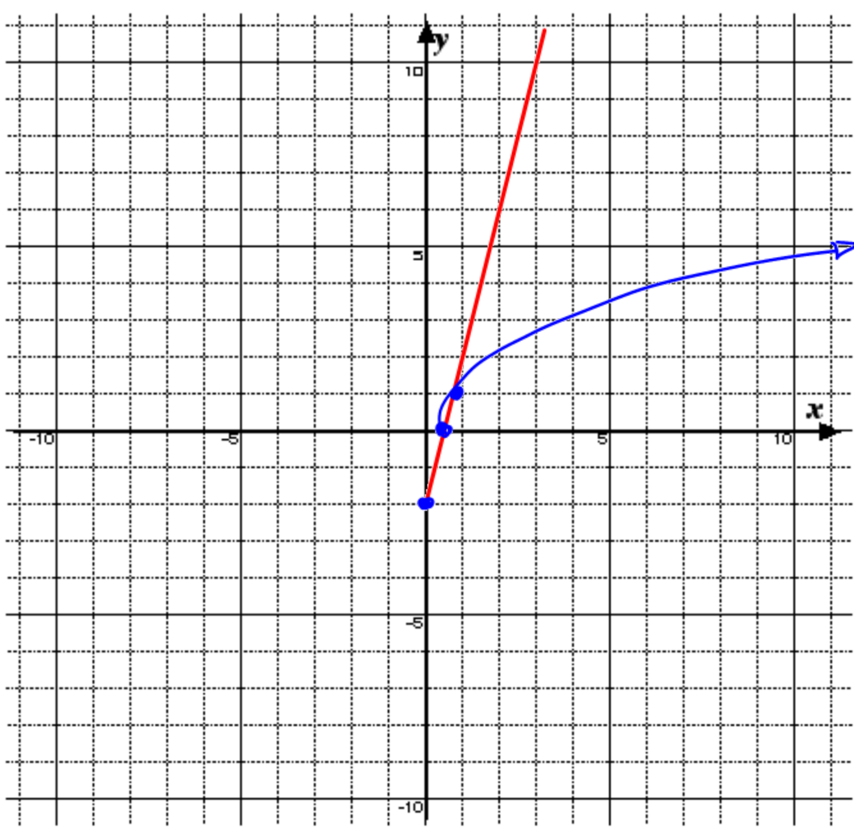
Example 1: Linear Functions and their Square Roots

Given $y = 4x - 2$ and $y = \sqrt{4x-2}$, complete the following:

a) Complete the table of values for both functions.

x	$y = 4x - 2$	$y = \sqrt{4x-2}$
0.5	0	0
0.75	1	1
1	2	1.4
1.5	4	2
2	6	2.5

b) Graph $y = 4x - 2$ and $y = \sqrt{4x-2}$ on the same grid. Identify the domain and range of each function and any invariant points.



$y = 4x - 2$

$0 = 4x - 2$

$2 = 4x$

$x = \frac{1}{2}$

domain

$x \geq \frac{1}{2}$

range

$y \geq 0$

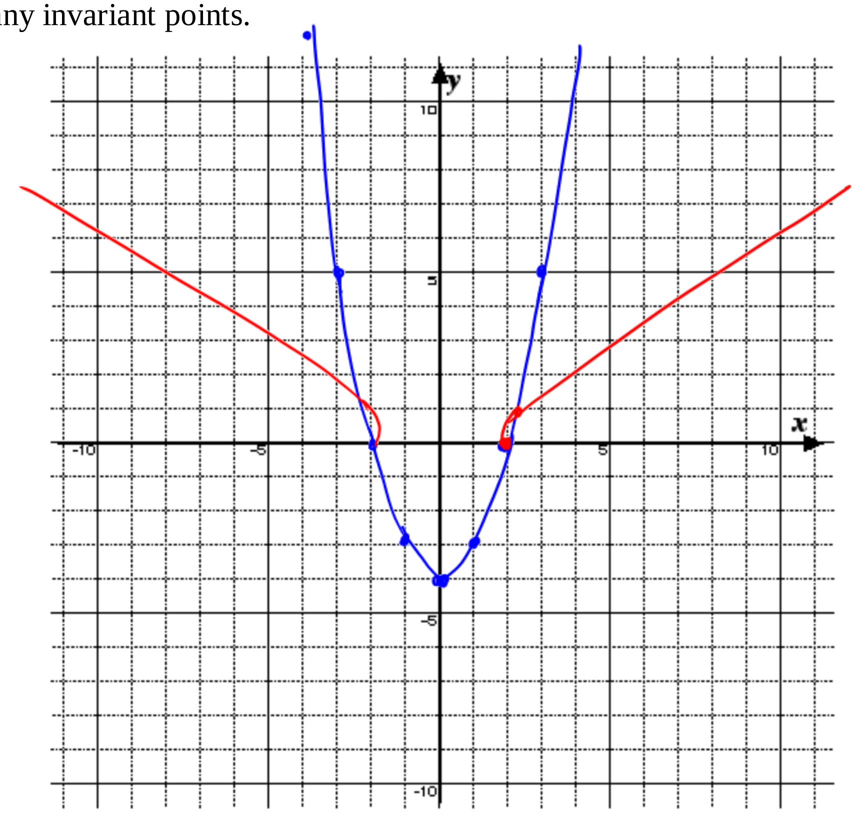
Example 2: Quadratic Functions and their Square Roots

Given $y = x^2 - 4$ and $y = \sqrt{x^2 - 4}$, complete the following. (NOTE: $\sqrt{x^2 - 4} \neq x - 2$!!)

a) Complete the table of values for both equations. (NOTE: The square root of a quadratic function CANNOT be found by simply transforming the equation $y = \sqrt{x}$.)

x	$y = x^2 - 4$	$y = \sqrt{x^2 - 4}$
–4	12	3.5
–3	5	2.2
–2	0	0
–1	-3	n.p
0	-4	n.p
1	-3	n.p
2	0	0
3	5	2.2
4	12	3.5

b) Graph $y = x^2 - 4$ and $y = \sqrt{x^2 - 4}$ on the same grid. Identify the domain and range of each function and any invariant points.



Example 3: The Square Roots of Other Quadratics

a) Graph $y = 4 - x^2$ and $y = \sqrt{4 - x^2}$ on the same grid. Determine the domain and range of each function and state the values of any invariant points.

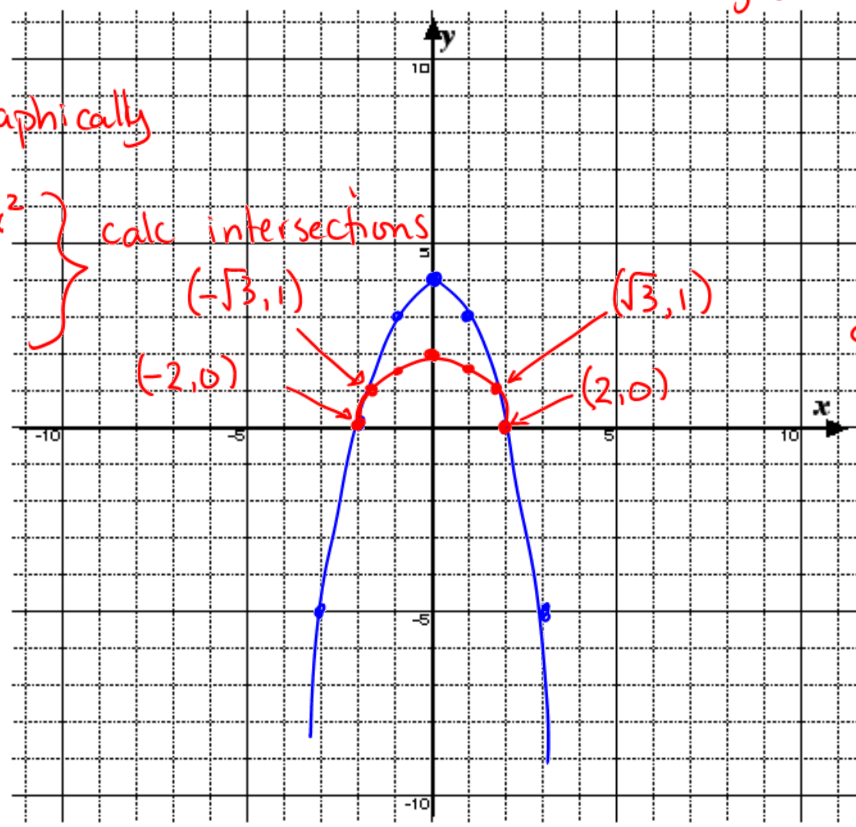
$y = -x^2 + 4$

graphically

$y_1 = \sqrt{4 - x^2}$

$y_2 = 1$

calc intersections



algebraically

$1 = \sqrt{4 - x^2}$

$1 = 4 - x^2$

$x^2 = 3$

$x = \pm\sqrt{3}$

$y = \sqrt{4 - x^2}$

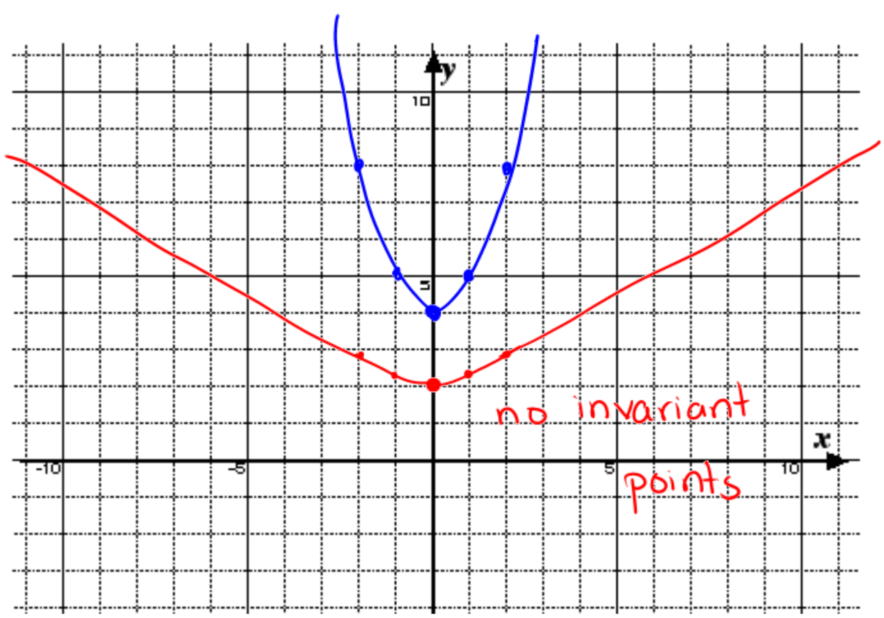
domain

$-2 \le x \le 2$

range

$0 \le y \le 2$

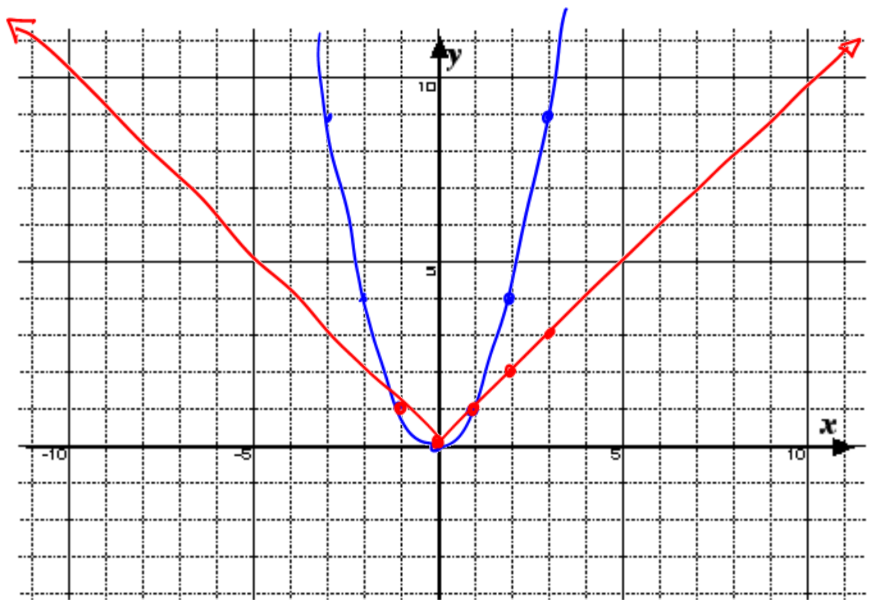
b) Graph $y = x^2 + 4$ and $y = \sqrt{x^2 + 4}$ on the same grid. Determine the domain and range of each function and state the values of any invariant points.



domain: $x = \mathbb{R}$

range: $y \ge 2$

Example 4: Graph $y=\sqrt{x^2}$. Give its domain and range.



domain: $x = \mathbb{R}$

range: $y \leq 0$

What other function has the same graph as this? $y=\sqrt{x^2}$ is same as $y=|x|$

Does that mean that these are equivalent functions? Explain.

These appear to be equivalent. Transforming each function the same way keeps them coincidental

Are the graphs of $y=\left(\sqrt{x}\right)^2$ and $y=\sqrt{x^2}$ the same? Explain.

no

$y=\sqrt{x}$ has a restricted domain $x \geq 0$, so it is only half of the graph.

Note: All of the transformations studied in chapter 1 were when the function was operating on a linear function (ie. We compared the graphs of $y=f(x)$ with the graph of $y=a\,f(linear\ \ function)+d$ or $y=a\,f(bx+c)+d$)) We have now looked at what happens when we compare the graph of $y=\sqrt{x}$ with $y=\sqrt{quadratic\ \ function}$. This requires a completely different analysis, and cannot be in terms of translations, reflections or stretches.