The Antiderivative

The Indefinite Integral

Recall that the function $F(x)$ is an antiderivative of the function $f(x)$ if it has the property that $\frac{dF(x)}{dx} = f(x)$ or $F'(x) = f(x)$. Also, if $F(x)$ is an antiderivative of $f(x)$, then so is $F(x)+C$. The set of all antiderivatives of a function $f(x)$ is called the indefinite integral of *f* with respect to *x* and is denoted by

$$\int f(x)\,dx$$

the symbol $\int$ is integral symbol → "antidifferentiation"

the function *f* is the integrand.

x is the variable of integration

Thus this notation simply means to find all antiderivatives of the function $f(x)$. By the Mean Value Theorem, we saw that

$$\int f(x)\,dx = F(x)+C$$

This is read as the indefinite integral of $f(x)$ with respect to *x* is $F(x)+C$. The constant *C* is called the constant of integration and must be included in your answer. Once $F(x)+C$ is found, we have ***integrated*** or ***evaluated*** the integral.

Examples

1. Evaluate $\int 6x\,dx$ $= 3x^2 + C$

How does this answer compare to $\int 6t\,dt$? $= 3t^2 + C$

2. Evaluate: $\int \frac{1}{\sqrt[3]{x}}\,dx$ $= \int x^{-\frac{1}{3}}\,dx = \frac{3}{2}x^{\frac{2}{3}} + C$

3. Evaluate:

$\int \cos x\,dx = \sin x + C$

check by differentiating

$\int \cos 2x\,dx = \frac{1}{2}\sin(2x) + C$

$y' = \frac{1}{\cancel{2}}\cos(2x)\cdot\cancel{2}$ ✓

$\int \cos 3x\,dx = \frac{1}{3}\sin(3x) + C$

$\int \cos kx\,dx = \frac{1}{k}\sin(kx) + C$

4. Evaluate:

$\int e^x\,dx = e^x + C$

check.

$\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$

$y' = \frac{1}{\cancel{2}}e^{2x}\cdot(\cancel{2})$ ✓

$\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$

$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

5. Evaluate: $\int\left(3x^2 + 7x - 5\right)dx$ $= x^3 + \frac{7}{2}x^2 - 5x + C$