1.8 Slopes of secants and tangents

For each of the following, write an expression for the slope of the secant through the given point. Then determine the slope of the tangent at the given point.

1. $y = x^2 + 5x$ at $x = 3$ $m_{sec} = \frac{f(3+h) - f(3)}{h}$ $= \frac{(3+h)^2 + 5(3+h) - 24}{h}$ $= \frac{9+6h+h^2+15+5h-24}{h} = \frac{11h+h^2}{h}$ $m_{sec} = 11+h$ $m_{tan} = \lim_{h\to 0} = 11$	2. $y = -2x + 9$ at $x = 5$ $m_{sec} = \frac{f(5+h) - f(5)}{h}$ $= \frac{-10-2h+9 - (-1)}{h}$ $= \frac{-2h}{h}$ at $x=5$ $m=-2$
3. $y = 7$ at $x = -3$ $m=0$ horizontal line.	4. $y = x^3$ at $x = 4$ $m_{sec} = \frac{(4+h)^3 - 4^3}{h}$ $= \frac{64+48h+12h^2+h^3-64}{h}$ $= \frac{48h+12h^2+h^3}{h}$ $m_{sec} = 48+12h+h^2$ $m_{tan} = 48$
5. $y = \sqrt{2x+3}$ at $x = 1$ $m_{sec} = \frac{\sqrt{2(1+h)+3} - \sqrt{2(1)+3}}{h}$ $= \frac{\sqrt{5+2h} - \sqrt{5}}{h} \cdot \frac{\sqrt{5+2h}+\sqrt{5}}{\sqrt{5+2h}+\sqrt{5}}$ $= \frac{5+2h-5}{h(\sqrt{5+2h}+\sqrt{5})}$ $= \frac{2}{\sqrt{5+2h}+\sqrt{5}}$ $m_{tan} = \frac{2}{2\sqrt{5}}$ or $\frac{1}{\sqrt{5}}$	6. $y = \frac{2}{x-3}$ at $x = 4$ $m_{sec} = \frac{\frac{2}{4+h-3} - \frac{2}{4-3}}{h}$ $= \frac{\frac{2}{1+h} - \frac{2}{1}}{h}$ $= \frac{2-2(1+h)}{(1+h)(1)} \cdot \frac{1}{h} = \frac{-2\cancel{h}}{\cancel{h}(1+h)} = \frac{-2}{1+h}$ $\boxed{m_{tan} = -2}$
7. $y = x^2 + 9$ at $x = a$ $m_{sec} = \frac{(a+h)^2 + 9 - (a^2+9)}{h}$ $= \frac{a^2+2ah+h^2+9-a^2-9}{h}$ $= \frac{2ah+h^2}{h} = \frac{h(2a+h)}{h}$ $m_{sec} = 2a+h$ $\boxed{m_{tan} = 2a}$	8. $y = f(x)$ at $x = a$ $m_{sec} = \frac{f(a+h) - f(a)}{h}$ $m_{tan} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$