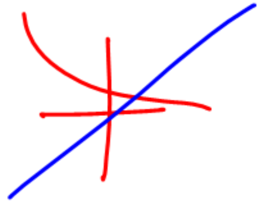
1.5 Warmup

Determine the following limits: $y = e^{-x}$



1) $\lim\limits_{x\to\infty}\dfrac{e^{-x}}{x} = 0$

2) $\lim\limits_{x\to-\infty}\dfrac{e^{-x}}{x} = -\infty$ ↙ grows much faster than $y=x$

3) $\lim\limits_{x\to 2}\dfrac{\sqrt{x+7}-1}{x-2} = \dfrac{2}{0}$ ∴ asymptote

$x \to 2^+$ $\dfrac{+}{+} = +\infty$

$x \to 2^-$ $\dfrac{+}{-} = -\infty$

<u>no limit</u>

4) $\lim\limits_{x\to\infty}\dfrac{\sin x}{x} =$ numerator that is $-1 \le y \le 1$

$= 0$ because your fraction is infinitely small, doesn't matter if $\sin x$ is $+$ or $-$

5) $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{x} = e$

the definition for e

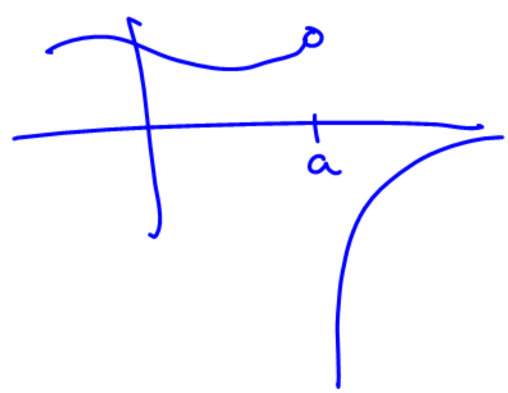
6) $\lim\limits_{x\to\infty}\tan x$

-no limit

as graph gets to ∞ the value of $\tan x$ does not approach a single value

7) For the function $y = f(x)$, you are told that $\lim\limits_{x\to a^+} f(x) = -\infty$. What does this tell you about the function? there is an asymptote at $x = a$

8) For the previous question, what is $\lim\limits_{x\to a^-} f(x)$?



no it could be anything.

ex. it could be part of a piecewise function

Limits of Trigonometric Functions

What is $\lim_{x \to 0} \frac{\sin x}{x}$?

Numerical Approach

Approaching x from the right

x	$\frac{\sin x}{x}$
0.2	.99335
0.1	.99833
0.01	.9998
0.001	1

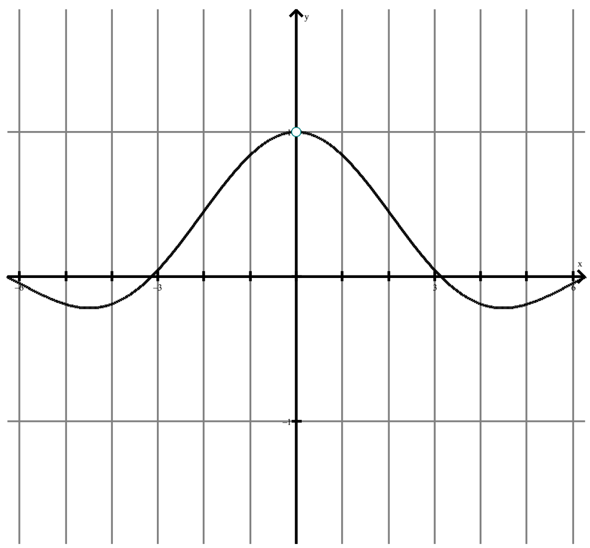
Approaching x from the left

x	$\frac{\sin x}{x}$
−0.2	.99335
−0.1	.99833
−0.01	.9998
−0.001	1

The table suggests that $\lim_{x \to 0} \frac{\sin x}{x} = $ 1

Graphical Approach

Graphing $y = \frac{\sin x}{x}$ yields the graph:



The graph seems to suggest that:

$\lim_{x \to 0^-} \frac{\sin x}{x} = 1$

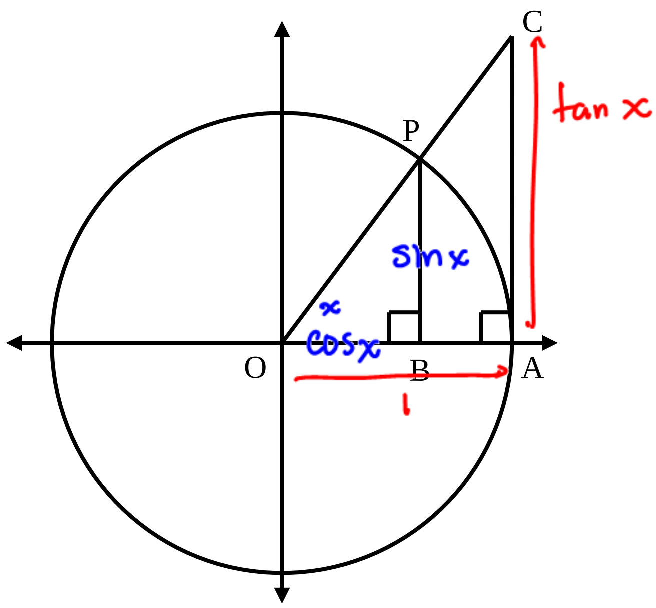
$\lim_{x \to 0^+} \frac{\sin x}{x} = 1$

Thus the graph seems to suggest that $\lim_{x \to 0} \frac{\sin x}{x} = $ 1

To prove this geometrically requires the use of the Sandwich Theorem (Yummy, Yummy - also known as the Squeeze Theorem)

$$\text{If } g(x) \le f(x) \le h(x) \text{ and } \lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L, \text{ then } \lim_{x \to a} f(x) = L$$

Geometric approach



A of Δ OPB $\leq$ pie POA $\leq$ A of Δ COA

$$\left[\frac{\cos x \cdot \sin x}{2} \leq \pi \cdot \frac{x}{2\pi} \leq \frac{1 \cdot \tan x}{2}\right] 2$$

$$\left[\cos x \sin x \leq x \leq \frac{\sin x}{\cos x}\right] \frac{1}{\sin x}$$

$$\cos x \leq \frac{x}{\sin x} \leq \frac{1}{\cos x}$$

$$\frac{1}{\cos x} \geq \frac{\sin x}{x} \geq \cos x$$

as $x \to 0$ $\quad 1 \geq \frac{\sin x}{x} \geq 1$

Thus $\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$ provided x is measured in radians

Evaluate:

8) $\lim_{x\to 0} \frac{\sin x}{3x} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{3}$

$= (1)\left(\frac{1}{3}\right)$

$= \frac{1}{3}$

2) $\lim_{x\to 0} \frac{\sin(5x)}{x} = \lim_{x\to 0} \frac{\sin(5x)}{x} \cdot \frac{5}{5}$

$= \lim_{x\to 0} \frac{\sin(5x)}{5x} \cdot \frac{5}{1}$

$= (1)(5)$

$= 5$

3) $\lim_{x\to 0} \frac{\sin 3x}{\sin 5x} = \lim_{x\to 0} \frac{\sin 3x}{1} \cdot \frac{1}{\sin 5x}$

$\lim_{x\to 0} \frac{\sin 3x}{3x} \cdot 3\cancel{x} \cdot \frac{5x}{\sin 5x} \cdot \frac{1}{5\cancel{x}}$

$= (1)(3)(1) \cdot \left(\frac{1}{5}\right)$

$= \frac{3}{5}$

4) $\lim_{x\to 0} \frac{\sin x}{x^2+5x} = \lim_{x\to 0} \frac{\sin x}{x(x+5)}$

$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{x+5}$

$= (1) \left(\frac{1}{5}\right)$

$= \frac{1}{5}$

5) $\lim_{x\to 0} \frac{\sin x \cos x}{x^2-7x}$

$\lim_{x\to 0} \frac{\sin x \cdot \cos x}{x(x-7)}$

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\cos x}{x-7}$

$= (1) \cdot \frac{1}{-7}$

$= -\frac{1}{7}$

6) $\lim_{x\to 0} \frac{\sin^3 x}{x^2} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{x} \cdot \sin x$

$= (1) \cdot (1) \cdot (0)$

$= 0$

7) $\lim_{x\to 0} \frac{1-\cos^2 x}{x}$

$\lim_{x\to 0} \frac{\sin^2 x}{x}$

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \sin x$

$= (1)(0) = 0$

8) $\lim_{x\to 0} \frac{\cos x - 1}{x} = \lim_{x\to 0} \frac{-(1-\cos x)}{x} \frac{(1+\cos x)}{(1+\cos x)}$

$= \lim_{x\to 0} \frac{-(1-\cos^2 x)}{x(1+\cos x)}$

$= \lim_{x\to 0} \frac{-\sin^2 x}{x(1+\cos x)}$

$= -\frac{\sin x}{x} \cdot \frac{\sin x}{1+\cos x}$

$= -(1)\left(\frac{0}{2}\right)$

$= 0$

9) $\lim_{x\to \pi} \frac{\sin(x-\pi)}{x-\pi} = 1$

$x \to \pi$

$x - \pi \to 0$

let $a = x - \pi$

$\lim_{a\to 0} \frac{\sin(a)}{a} = 1$