Functions And Their Graphs

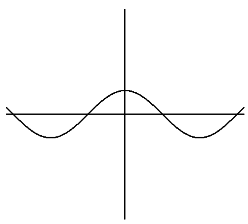
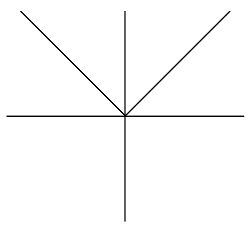
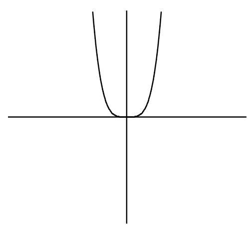
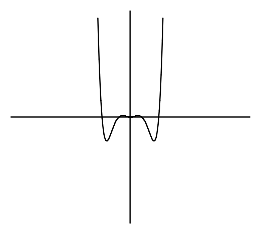
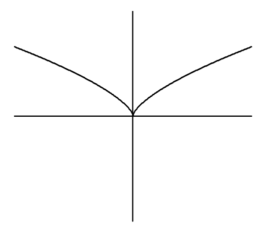
Definition of a Function

A function is a relation which assigns to each value in the domain only one value in the range. Graphically, a relation is a function if it passes the vertical line test.

Even and Odd Functions

A function is **even** if it has the property $f(x) = f(-x)$

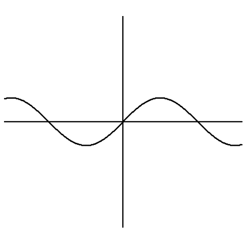
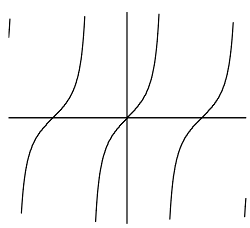
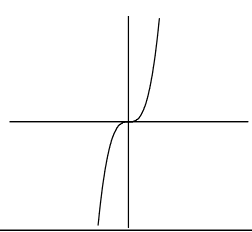
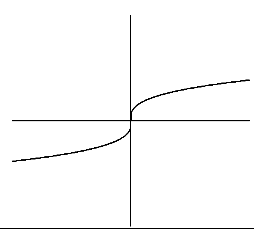
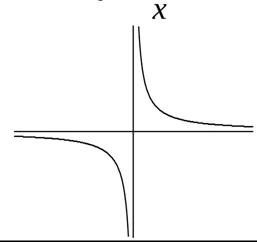
Some examples of even functions are:

$y = \cos x$	$y = \lvert x \rvert$	$y = x^4$	$y = 3x^6 - 5x^4 + x^2$	$y = x^{\frac{2}{3}}$
				

The graphs of these functions possess an axis of symmetry at x=0

A function is **odd** if it has the property $f(x) = -f(-x)$

Some examples of odd functions are: reflected both vertically and horizontally.

$y = \sin x$	$y = \tan x$	$y = 2x^3$	$y = \sqrt[3]{x}$	$y = \frac{1}{x}$
				

The graphs of these functions possess rotational symmetry.

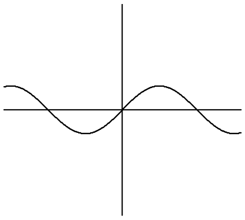
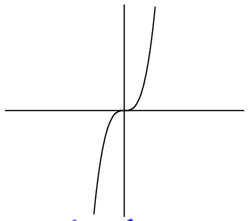
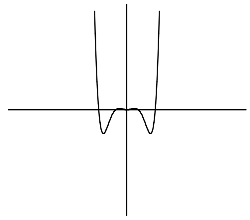
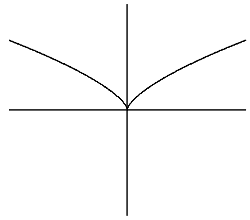
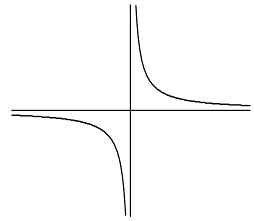
Knowing that a function is even or odd means that once you know how the graph behaves for $x \geq 0$, you can then determine what the other half of the graph looks like.

One-to-One Functions

A function is one-to-one if each element in the range corresponds with only one element in the domain.

- passes a vertical line test
- passes a horizontal line test.

Which of the following functions are one-to-one?

$y = \sin x$	$y = 2x^3$	$y = 3x^6 - 5x^4 + x^2$	$y = x^{\frac{2}{3}}$	$y = \frac{1}{x}$
				
no	yes	no	no	yes.

To test whether a function is one-to-one, use horizontal line test.

If a function is one-to-one, then

1) inverse is also a function

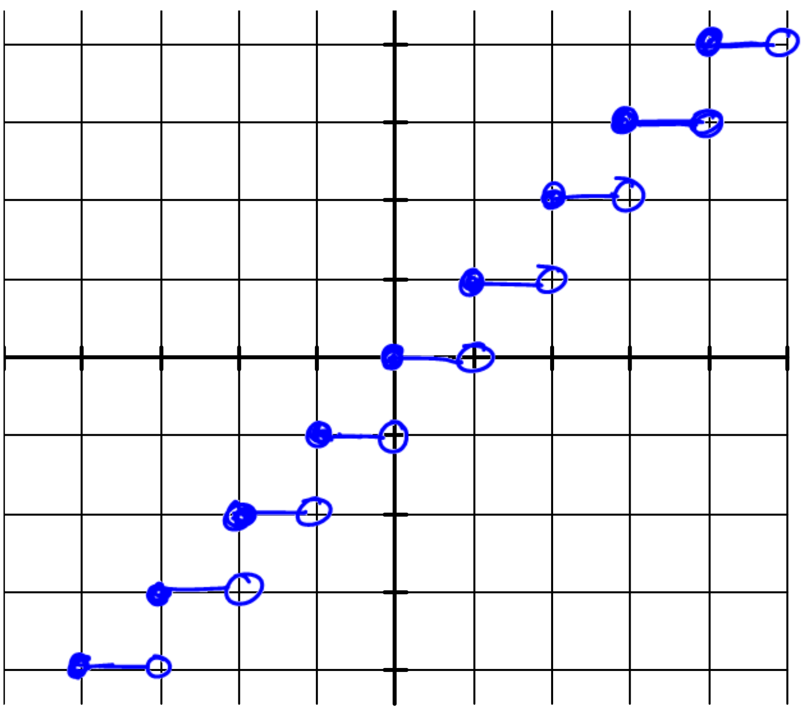
2) each output corresponds to a unique input.

The Greatest Integer Function: $y = \text{int}(x)$ *or* $y = [x]$

Definition: $\text{int}(x) =$ the greatest integer that is less than or equal to x

$\text{int}(3) =$ 3	$\text{int}(3.1) =$ 3	$\text{int}(3.9) =$ 3	$\text{int}(3.999) =$ 3	$\text{int}(4) =$ 4
$\text{int}(-3) =$ -3	$\text{int}(-3.1) =$ -4	$\text{int}(-3.9) =$ -4	$\text{int}(0) =$ 0	$\text{int}(-.1) =$ ___

The graph of $y = \text{int}(x)$:



(0,0)

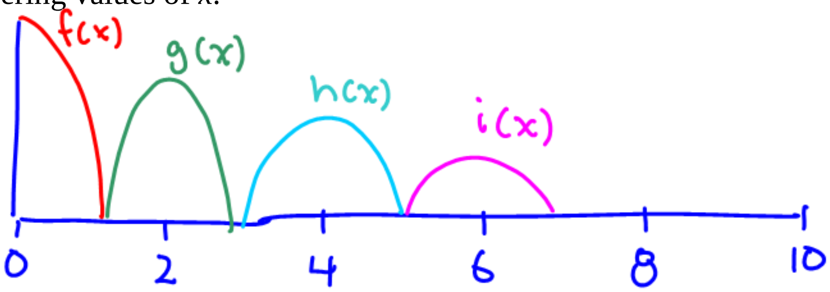
(0.5,0)

(0.999999,0)

(1,1)

Piecewise Functions

Sometimes a function may not be easily represented as a single function, but is instead composed of several "pieces" of different functions on different parts of its domain. A good example of a situation where this might occur is a bouncing ball. It could probably be represented as different parabolas for differing values of *x*.

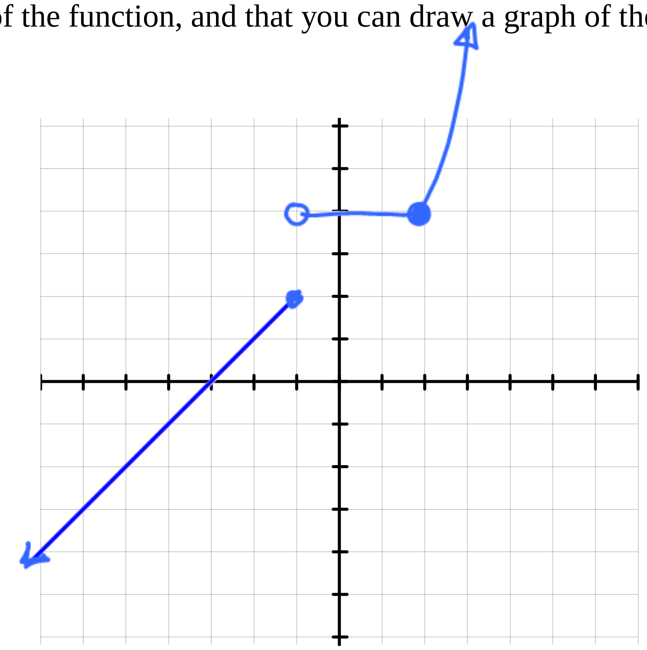


$$y = \begin{cases} f(x) & 0 \le x < 1 \\ g(x) & 1 \le x < 3 \\ h(x) & 3 \le x < 5 \\ i(x) & 5 \le x < 7 \end{cases}$$

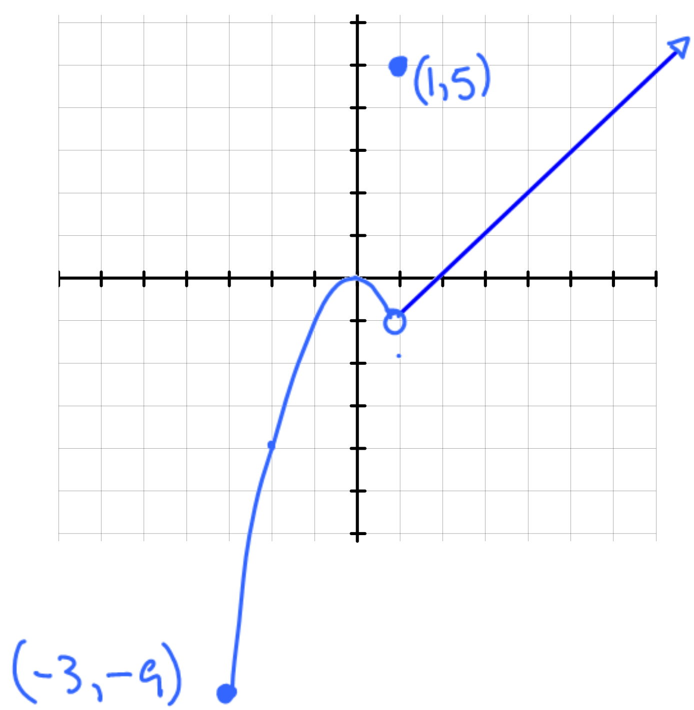
It is important that you understand the definition of the function, and that you can draw a graph of the function.

Graph the following functions

1) $y = \begin{cases} x+3 & x \le -1 \\ 4 & -1 < x < 2 \\ x^2 & x \ge 2 \end{cases}$

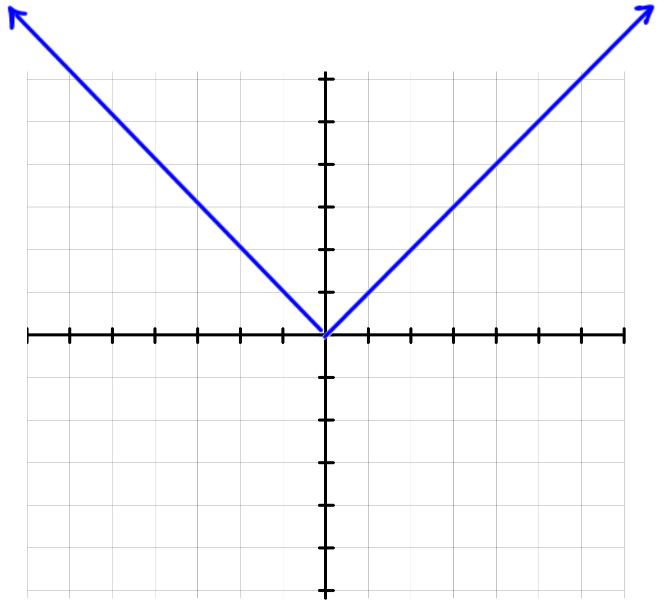


2) $y = \begin{cases} -x^2 & -3 \le x < 1 \\ 5 & x = 1 \\ x-2 & x > 1 \end{cases}$



3) Give a piecewise definition for $y = |x|$

$$y = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



4) Graph the function $y = \dfrac{|x-1|}{x-1}$. Give a piecewise definition for the function.

$$y = \begin{cases} 1 & x > 1 \\ \text{n.p} & x = 1 \\ -1 & x < 1 \end{cases}$$