11.3B Warmup

1. Expand: $(2x-y)^4$

2. Express the expansion of $(2x-y)^4$ using combinations.

$a=2x$, $b=-y$

$(a+b)^4$

$${}_4C_0(a)^4 + {}_4C_1a^3b + {}_4C_2a^2b^2 + {}_4C_3ab^3 + {}_4C_4b^4$$

$${}_4C_0(2x)^4 + {}_4C_1(2x)^3(-y) + {}_4C_2(2x)^2(-y)^2 + {}_4C_3(2x)(-y)^3 + {}_4C_4(-y)^4$$

3. Simplify: $(a+2b)^3-(a-2b)^3$

4. How many terms are there in the expansion of $(2x-y)^{15}$?

5. What would the 10^{th} term in the expansion of $(2x-y)^{15}$ be?

11.3B The Binomial Theorem

Visualizing – The Binomial Expansion of $(a + b)^4$

$$(a+b)^4 = {}_4C_0\, a^4b^0 + {}_4C_1\, a^3b^1 + {}_4C_2\, a^2b^2 + {}_4C_3\, a^1b^3 + {}_4C_4\, a^0b^4$$

0 *b*'s, 1 *b*, 2 *b*'s, 3 *b*'s, 4 *b*'s

Number of ways to choose this many b's from 4 factors of $(a + b)$

The Binomial Theorem (using combinations)

For any whole number *n*:

$$(a+b)^n = {}_nC_0\, a^n\, b^0 + {}_nC_1\, a^{n-1}\, b^1 + {}_nC_2\, a^{n-2}\, b^2 + \ldots + {}_nC_k\, a^{n-k}\, b^k + \ldots + {}_nC_n\, a^0\, b^n$$

1st term, 2nd term, 3rd term, _____ term, $(n+1)^{th}$ term

For this expansion the general term is: $t_{k+1} =$ $_nC_k\,(a)^{n-k}(b)^k$

Example 1: Write the first four terms of the binomial expansion of $(x + 2y)^{12}$.

1st term: $t_{k+1} = t_1$, $k = 0$

$t_1 = {}_{12}C_0\,(x)^{12-0}(2y)^0$

2nd term: $k = 1$

${}_{12}C_1\,(x)^{12-1}(2y)^1$

3rd term: $k = 2$

${}_{12}C_2\,(x)^{12-2}(2y)^2$

4th term: $k = 3$

${}_{12}C_3\,(x)^{12-3}(2y)^3$

Example 2: Determine the 5th term in the expansion of $(x-3)^9$.

$k = 4$

$t_{k+1} = {}_nC_k\,(x)^{n-k}(y)^k$

$t_5 = {}_9C_4\,(x)^5(-3)^4$

Example 3 Find the coefficient on x^3 in the expansion of $(1 - 2x)^{12}$

$t_{k+1} = {}_nC_k\,(1)^{n-k}(-2x)^k$

$= {}_{12}C_3\,(1)^{12-3}(-2x)^3$

$= 220\,(1)^9(-8x^3)$

$= -1760x^3$

Example 4 One term in the expansion of $(x + a)^8$ is $448x^6$. Determine the value of a.

${}_nC_k\,(x)^{n-k}(a)^k = 448x^6$

${}_8C_2\,(x)^6(a)^2 = 448x^6$

$28\,x^6 \cdot a^2 = 448x^6$

$28a^2\,x^6 = 448x^6$

$28a^2 = 448$

$a^2 = 16$

$a = \pm 4$

Example 5 The 4th term in the expansion of $\left(x - \frac{1}{2}\right)^n$ is $-15x^7$. Determine n.

$k = 3$

${}_nC_k\,(x)^{n-k}\left(-\frac{1}{2}\right)^k$

${}_nC_3\,(x)^{n-3}\left(-\frac{1}{2}\right)^3$

$n - 3 = 7$

$n = 10$

Example 6 Determine the constant term in the expansion of $\left(x^2 - \frac{1}{x}\right)^6$

${}_6C_k\,(x^2)^{6-k}\left(-\frac{1}{x}\right)^k$

${}_6C_4\,(x^2)^2\left(-\frac{1}{x}\right)^4$

$15\,(x^4)\left(\frac{1}{x^4}\right)$

constant term = 15

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Combinatorics Review Warmup

1. How many arrangements of the letters in the word SCHOOLS are there if

a) there are no restrictions	b) the letter S must be at the end	c) the two O's must be together

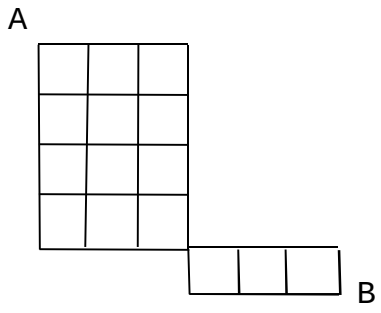
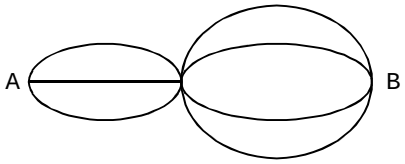
2. How many five card hands are possible from a standard 52 card deck

a) containing exactly 3 clubs	b) containing 2 jacks, 2 aces and one other card	c) containing exactly 4 red cards	d) containing at least 3 red cards

3. Solve for *n*:

a) ${}_nP_2 = 56$	b) $\frac{n!}{80} = (n-1)!$	c) ${}_nC_2 = 28$

4. How many routes from A to B if you are always moving to get closer to B?

Combinatorics Review

1. Simplify: $\dfrac{n(n+1)!}{(n-1)!}$

2. In the expansion of $(2a-3b)^6$, find the coefficient of the term containing a^4b^2

3. Solve for n: $\dfrac{(n-1)!}{(n-3)!}=30$

4. A coach must choose 3 out of 10 players for a tie-breaking penalty shot. If the coach must designate the order of the 3 players, how many arrangements are possible?

5. Determine the fourth term in the expansion $(x-2y)^5$

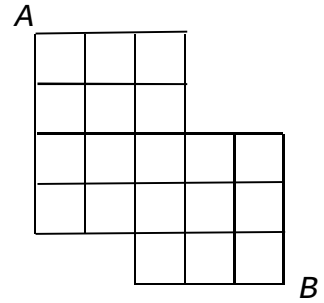
6. Express ${}_{33}C_5$ in factorial notation

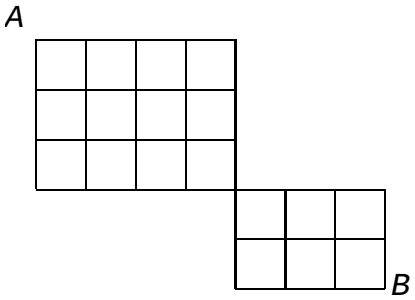
7. There are 10 boys and 20 girls in a class. How many committees of 3 are possible if a committee contains 1 boy and 2 girls?

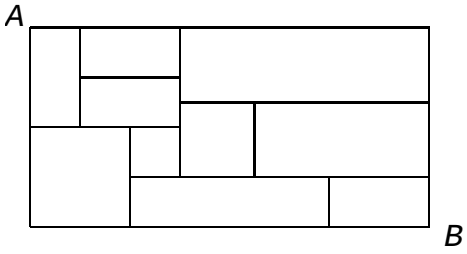
8. How many ways can a committee of 3 people be selected from a class of 30? How many ways can an executive committee (Pres., Vice Pres, Secretary) be selected from this class?

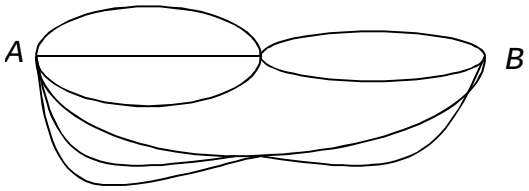
9. A toy box has 5 different cars and 6 different trucks.
   a) How many ways can 5 toys be chosen if there are 2 cars and 3 trucks?
   b) How many ways can 5 toys be chosen if there are at least 3 cars?

10. Assuming that you are always moving closer to *B*, how many paths are there from *A* to *B*?









11. What is the 5th number in the 27th row of Pascal's triangle?

12. Given TSAWWASSEN, how many permutations are there
    a) without restrictions
    b) if the first letter must be S
    c) the three S's are together

13. Find the 7th term in the expansion $(3x-2y)^{15}$

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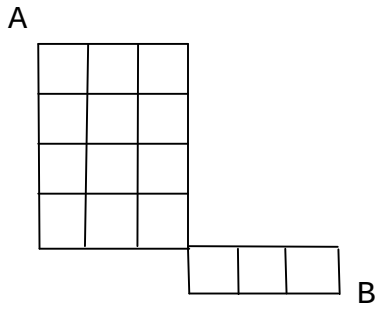	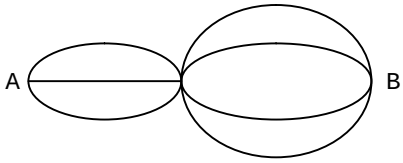	

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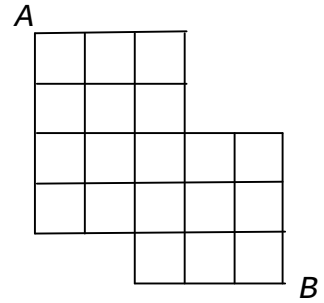

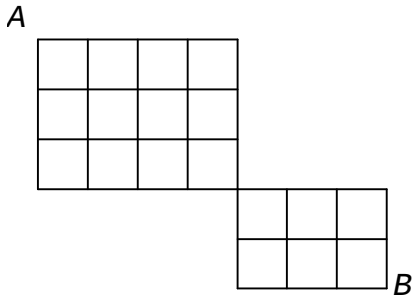

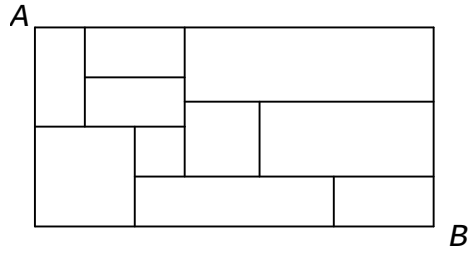

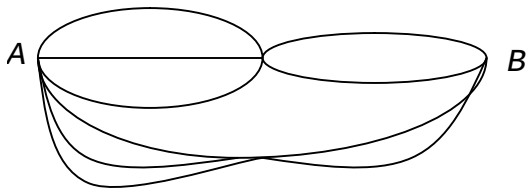

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