Trigonometric Integrals

Integrals you need to know:

$\int \sin kx \ dx = -\frac{1}{k}\cos kx + C$ $\qquad$ $\int \cos kx \ dx = \frac{1}{k}\sin kx + C$

$\int \sec^2 kx \ dx = \frac{1}{k}\tan kx + C$ $\qquad$ $\int \csc^2 kx \ dx = -\frac{1}{k}\cot kx + C$

Determine the following:

1. $\int \sin 5x \ dx = -\frac{1}{5}\cos 5x + C$

2. $\int (\cos^2\theta - \sin^2\theta)\, d\theta = \int \underbrace{\cos 2\theta}_{\cos 2\theta}\, d\theta = \frac{1}{2}\sin 2\theta + C$

3. $\int \frac{2}{3} x \sin(3x)^2 \ dx$

$(3x)^2 = 9x^2$ $\qquad$ $u = 9x^2$

$\frac{du}{dx} = 18x$ $\qquad$ $dx = \frac{1}{18x}\,du$

$= \int \frac{2}{3}x \sin u \cdot \frac{1}{18x}\, du = \int \frac{1}{27}\sin u \, du$

$= -\frac{1}{27}\cos u + C$

$= -\frac{1}{27}\cos(3x)^2 + C$

4. $\int \frac{5\cos\sqrt{x}}{\sqrt{x}} \ dx = \int \frac{5\cos\sqrt{x}}{\sqrt{x}} \cdot 2\sqrt{x}\, du$

$\sqrt{x} = u$ $\qquad$ $u = x^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$ $\qquad$ $dx = 2\sqrt{x}\, du$

$= \int 10\cos u \, du = 10\sin u + C$

$= 10\sin\sqrt{x} + C$

5. $\int \sin^2 x \cos x \ dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$ $\qquad$ $dx = \frac{1}{\cos x}\, du$

$= \int u^2 \cdot \cos x \cdot \frac{1}{\cos x}\, du = \frac{1}{3}u^3 + C$

$= \frac{1}{3}\sin^3 x + C$

6. $\int \sec x \tan x \, dx$ $= \sec x + C$

I ♡ u, formula sheet!

7. $\int \csc x \cot x \, dx$ $= -\csc x + C$

8. $\int \frac{1}{\cos^2(3x+1)} \, dx$ $= \int \frac{1}{\cos^2 u} \cdot \frac{1}{3} \, du$

$u = 3x+1$

$\frac{du}{dx} = 3$

$dx = \frac{1}{3} du$

$= \int \sec^2 u \cdot \frac{1}{3} \, du$

$= \frac{1}{3} \tan u + C$

$= \boxed{\frac{1}{3} \tan(3x+1) + C}$

9. $\int \frac{\sin(\theta+2)}{\cos^2(\theta+2)} \, d\theta$

$u = \cos(\theta+2)$

$\frac{du}{d\theta} = -\sin(\theta+2) \cdot$

$d\theta = \frac{-1}{\sin(\theta+2)} du$

$= \int \frac{\sin(\theta+2)}{u^2} \cdot \frac{-1}{\sin(\theta+2)} \, du$

$= \int -\frac{1}{u^2} \, du$

$= \int -u^{-2} \, du$

$= u^{-1} + C$

$= \frac{1}{\cos(\theta+2)} + C$

or $\int \tan(\theta+2) \cdot \sec(\theta+2) \, d\theta$

$u = \theta+2$

$\frac{du}{d\theta} = 1$

$d\theta = du$

$\int \tan u \sec u \, du$

$\sec u + C$

or $\sec(\theta+2) + C$

10. $\int \sin^2 4x \, dx$