7.1 Characteristics of Exponential Functions

An exponential function is a function that can be written in the form $y = c^x$ where c is a constant ($c > 0$) and x is a variable.

Investigate the Graphs of Exponential Functions:

A) Let's graph $y = 2^x$ using a table of values.

x	–3	–2	–1	0	1	2	3	4
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16

Determine the following from your graph:

1) Domain $x \in \mathbb{R}$

2) Range $y > 0$

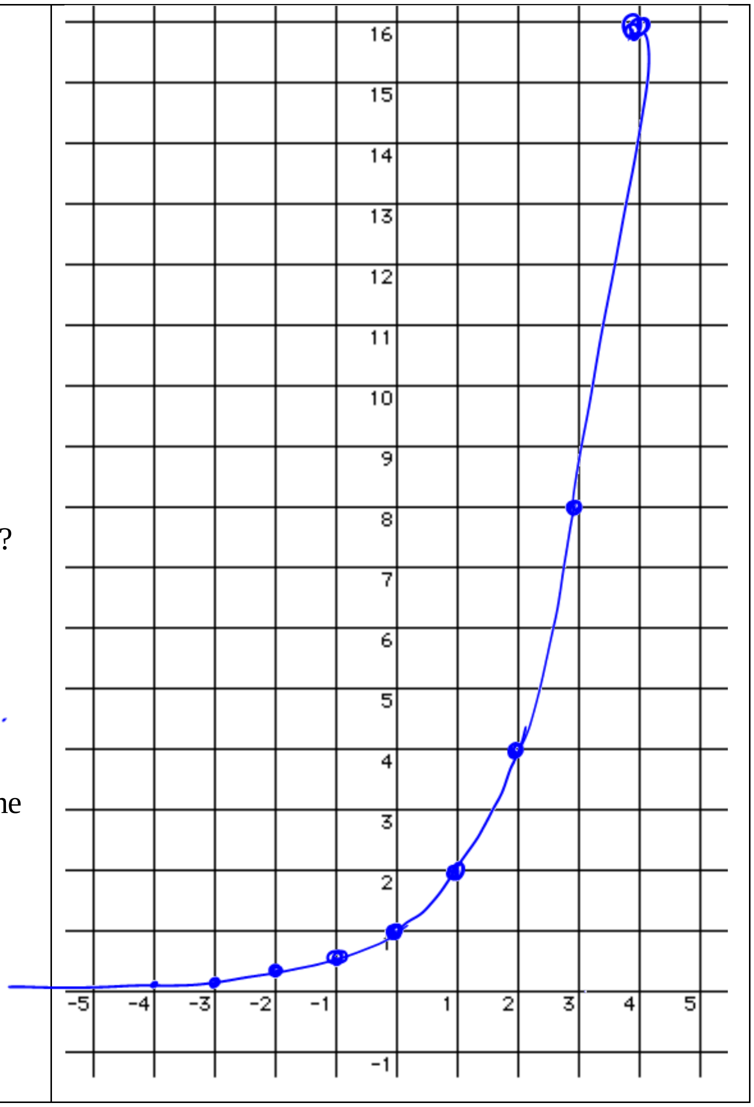
3) Intercept(s) y-int $= 1$

4) Asymptote(s) horizontal $y = 0$

5) What is the trend of the graph from left to right?
increasing "growth"

6) What do we call this type of exponential function?
exponential growth.

7) How would the graph of $y = 5^x$ compare with the graph of $y = 2^x$?
it would grow much faster.



B) Let's graph the function $y = \left(\frac{1}{2}\right)^x$ using a table of values. $y = (2^{-1})^x$

x	-3	-2	-1	0	1	2	3	4
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

Determine the following from your graph:

1) Domain $x \in \mathbb{R}$

2) Range $y > 0$

3) Intercept(s) y-int = 1

4) Asymptote(s) horizontal $y = 0$

5) What is the trend of the graph from left to right?

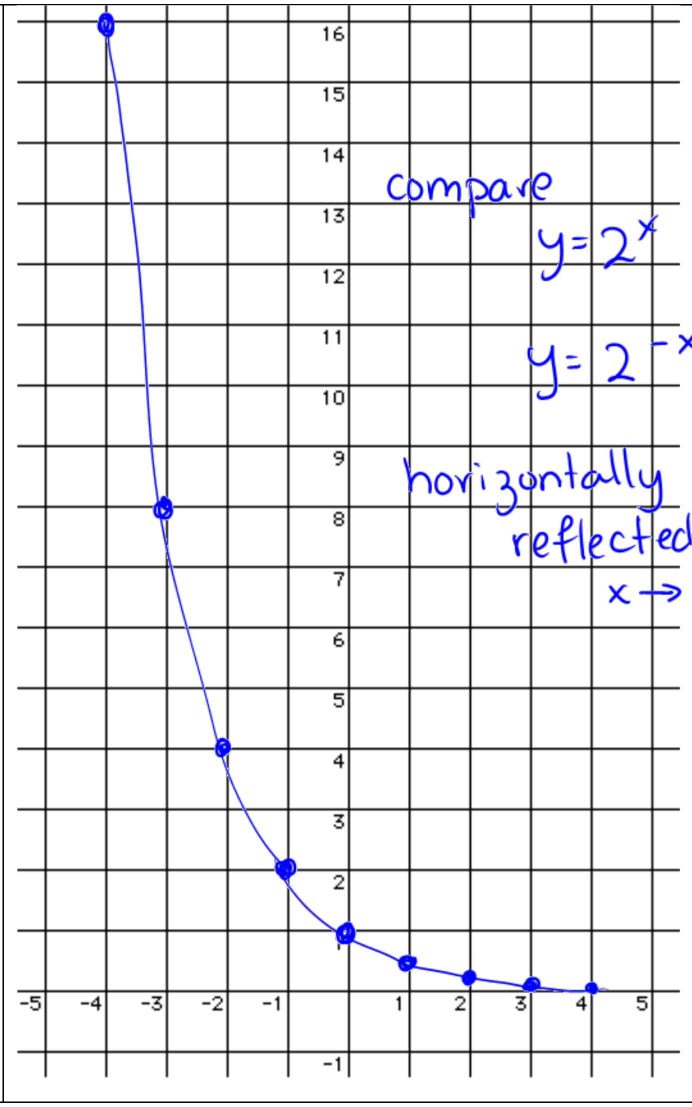
decreasing

6) What do we call this type of exponential function?

exponential decay.

7) How would the graph of $y = \left(\frac{1}{5}\right)^x$ compare with the graph of $y = \left(\frac{1}{2}\right)^x$?

decays a lot faster.



What do you think the graph of $y = (1)^x$ would look like?

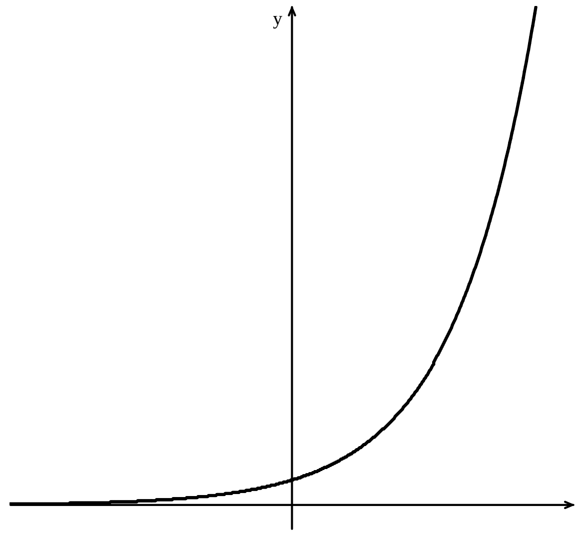
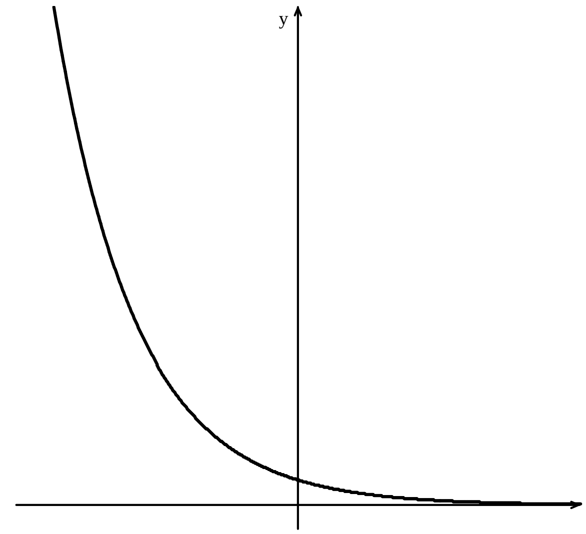
- horizontal line

What problems would you face if you tried to graph $y = (-2)^x$? for $x > 0$, switching between $+$ and $-$

x	0	1	2	3	4
y	1	-2	4	-8	16

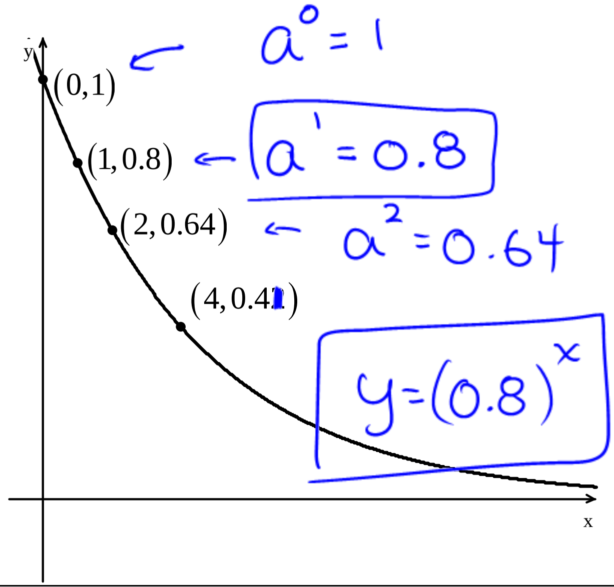
$(-2)^{\text{decimal}}$ = undefined for many values.

In summary, the graph of $y = c^x$ c must be a positive number $c \neq 1$

Exponential Growth $c > 1$	Exponential Decay $0 < c < 1$
	
eg $y = 2^x$	eg. $y = \left(\frac{1}{2}\right)^x$

Examples:

1) Determine the equation of the exponential function corresponding with each of the following graphs

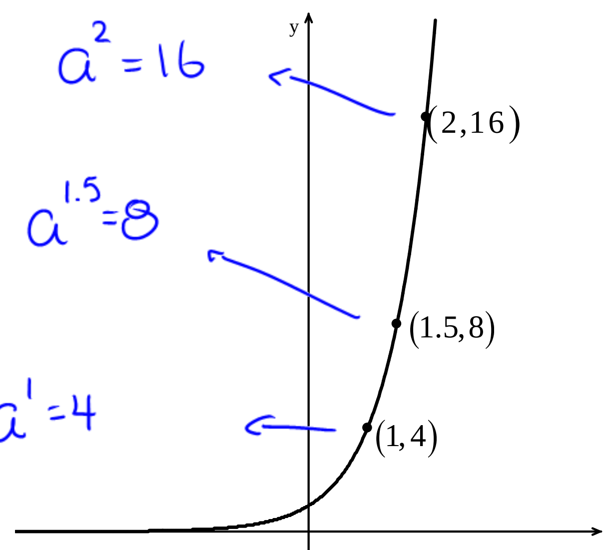


$a^0 = 1$

$\boxed{a^1 = 0.8}$

$a^2 = 0.64$

$\boxed{y = (0.8)^x}$



$a^2 = 16$

$a^{1.5} = 8$

$a^1 = 4$

$\boxed{y = 4^x}$

2) A scientist is doing an experiment on bacterial growth. She places a single bacterium in a petri dish and it divides to form two new bacteria within one hour. This process will continue as long as there is sufficient food and space for the bacteria.

a) Write an equation to illustrate this situation. What would the domain be?

$y = 2^x$

y: # of bacteria

x: # of hours.

domain $x \geq 0$

range $y \geq 1$, $y \in \mathbb{I}$

b) What is the population size after

3 hours?	3.5 hours?	4 hours?	24 hours?	48 hours?
$2^3 = 8$	$2^{3.5} = 11.3$ or ≈ 11 bacteria	$2^4 = 16$	16.8 million	2.8×10^{14} 281 trillion!

note: $2^{3.5}$ is not midway between 2^3 and 2^4

c) If the scientist added an antibiotic to the petri dish medium, the population of bacteria decreases at a rate of 7% per hour. What equation could you use to illustrate the relative size of the population over time? Approximately how many hours would have to pass so that the population is 50% of what it was to start with?

decrease by 7% per hour.

93% remains after 1 hour

x	y
1	.93
2	.86
4	.75
6	.65
8	.56
10	.48
9	.52

$y = (0.93)^x$

$50 = 100(0.93)^x$

$0.5 = 1(.93)^x$

$\boxed{\text{somewhere between 9 and 10 hours}}$