Warmup 5.2

Determine (An identity crisis is not a good thing to experience at this point)

a) $\int e^{15x}dx$

b) $\int\left(\frac{\sin 8x}{3}-x^{\frac{2}{3}}\right)dx$

c) $\int\left(\sin^2 x+\cos^2 x\right)dx$

d) $\int\left(\cos^2 x-\sin^2 x\right)dx$

e) $\int \cot^2 x\, dx$

Initial Value Problems

A differential equation is any equation containing a derivative.

$y' = 2x + 1$ $\qquad$ $3\frac{dy}{dx} = 4y + 2x$ $\qquad$ $2dy = (3x+2)dx$

An **initial value problem** is one where you are given the derivative and the value of the y at a given point, and are asked to find the original function. The value of y that is given to you is called the initial value. Solving the differential equation means to find **all** functions that satisfy the differential equation. Finding the solution that satisfies the initial condition means that we have solved the initial value problem.

Previous Problem	Initial Value Problem
Find the function $y = f(x)$ with a derivative of $\cos x + 3x$ and which passes through the point $(0, 5)$.	Solve the initial value problem: Differential equation: $\frac{dy}{dx} = \cos x + 3x$ Initial condition: $y(0) = 5$

Solve the initial value problems

1. $\frac{dy}{dx} = 9x^2 - 7x$, $\quad y(2) = 10$

$y = \int 9x^2 - 7x \; dx$

$y = 3x^3 - \frac{7}{2}x^2 + C$

$y(2) = 10$

$10 = 3(2)^3 - \frac{7}{2}(2)^2 + C$

$10 = 24 - 14 + C$

$C = 0$

$\boxed{y = 3x^3 - \frac{7}{2}x^2}$

2. $\frac{d^2y}{dt^2} = -2\cos t$, $\quad y(0) = 4$, $\quad y'(0) = -3$

$\frac{dy}{dt} = \int -2\cos t \; dt$

$\frac{dy}{dt} = -2\sin t + C$

$-3 = -2\sin(0) + C \qquad C = -3$

$\frac{dy}{dt} = -2\sin t - 3$

$y = \int -2\sin t - 3 \; dt$

$y = 2\cos t - 3t + C$

$y(0) = 4$

$4 = 2\cos(0) - 3(0) + C$

$4 = 2(1) + C$

$C = 2$

$\boxed{y = 2\cos t - 3t + 2}$

$\ln y \to y' \cdot \frac{1}{y}$

3. Solve the differential equations: a) $\frac{dy}{dt} = \underset{1}{y}$ b) $\frac{dy}{dt} = k\,y$

$\frac{dy}{y} = 1 \cdot dt$

$\int \frac{1}{y} \cdot dy = \int 1 \cdot dt$

$\ln y = t + C$

$\cancel{e}^{\cancel{\ln}} y = e^{t+c}$

$y = e^{t+c}$

$y = e^t \cdot \boxed{e^c}$ still a constant

$\boxed{y = k \cdot e^t}$

$\frac{dy}{y} = k \cdot dt$

$\int \frac{1}{y} \cdot dy = \int k \cdot dt$

$\ln y = kt + C$

$\cancel{e}^{\cancel{\ln}} y = e^{kt+c}$

$y = e^{kt+c}$

$y = e^{kt} \cdot \boxed{e^c}$

$\boxed{y = c \cdot e^{kt}}$

The solution to the differential equation, $\frac{dy}{dt} = ky$ is $y = c \cdot e^{kt}$

ie. The solution to $\frac{dy}{dt} = .05y$ is $y = c \cdot e^{.05t}$

4. An amount of money, y_0, is invested at 6.9 % compounded continuously. What does this mean?

y_0 = amount at time = 0

y = amount

$\frac{dy}{dt} = 0.069 \cdot y$

$\frac{dy}{dt}$ = rate at which amount is changing.

Model this situation as an initial value problem, and then determine the solution.

$\frac{dy}{dt} = 0.069 \cdot y$

$\boxed{y(0) = y_0}$

$\int \frac{dy}{y} = \int 0.069\, dt$

$\ln y = 0.069t + C$

$y = e^{0.069t + C}$

$y_0 = C \cdot e^{0.069(0)}$

$y_0 = C$

$\Rightarrow y = C \cdot e^{0.069t}$

$\boxed{y = y_0 e^{0.069t}}$