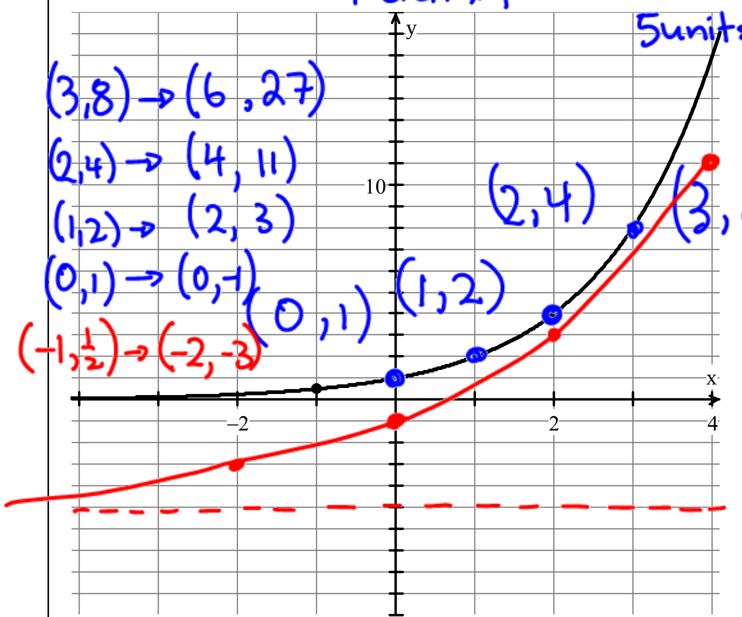
7.3 Warmup

1. Sketch a graph of the following exponential functions. Give domain, range, *y*-intercept, and equation of any asymptotes.

h. stretch by ×2

$y = 4(2)^{\frac{1}{2}x} - 5$

vertical stretch ×4 followed by 5 units down



Domain	Range
$x \in \mathbb{R}$	$y > {}^{-}5$
y-intercept	Asymptote
$(0,-1)$	$y = -5$

$y = 2a^x - b \qquad (a > b > 1)$

$b > 1$

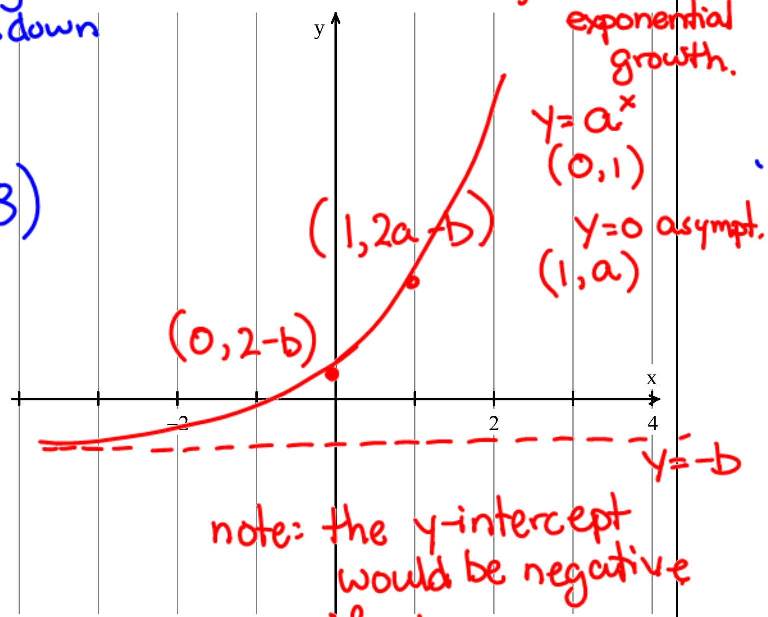
$a > b$, so $a > 1$ exponential growth.

$y = a^x$

$(0,1)$

$y = 0$ asympt.

$(1, a)$



note: the y-intercept would be negative if $b > 2$

Domain	Range
$x \in \mathbb{R}$	$y > -b$
y-intercept	Asymptote
$(0, 2-b)$	$y = -b$

2. Rewrite each of the following as a power with a base of 2.

32	$\sqrt{2}$	$\frac{1}{16}$	$\sqrt[5]{2^7}$	$\sqrt[5]{8^2}$
2^5	$2^{\frac{1}{2}}$	2^{-4}	$2^{\frac{7}{5}}$	$\sqrt[5]{(2^3)^2} = 2^{\frac{6}{5}}$
	* roots are fraction exponents	* fractions are negative exp.		

3. An radioactive element has a half-life of 300 years. Write an exponential function that will determine the percent remaining of a sample in *x* years?

$y = 100\left(\frac{1}{2}\right)^{\frac{x}{300}}$

$y = y_0 \cdot a^{\frac{x}{n}}$

actual time

time to do

initial

growth/decay factor

7.3 Solving Exponential Equations

Example 1. Solve the following exponential equations:

$$2^{3x} = 2^{5x-8}$$

$$3x = 5x-8$$
$$-2x = -8$$
$$x = 4$$

$$2^{3(4)} = 2^{5(4)-8}$$
$$2^{12} = 2^{12} \quad \checkmark$$

Property

$$a^x = a^y \iff x = y \quad \text{provided } a \neq 0, \pm 1$$

if the bases are equal then the exponents are equal

note: base $\neq 0$ or ± 1

Solve the following exponential equations by converting each side to the same base

$$8^{2x+1} = 16^{3x-5}$$

$8 = 2^3$ $\quad$ $16 = 2^4$

$$(2^3)^{2x+1} = (2^4)^{3x-5}$$
$$2^{3(2x+1)} = 2^{4(3x-5)}$$
$$3(2x+1) = 4(3x-5)$$
$$6x+3 = 12x-20$$
$$23 = 6x$$
$$\boxed{x = \frac{23}{6}}$$

$$9^{x-2} = \left(\frac{1}{27}\right)^{2x+1}$$

$9 = 3^2$ $\quad$ $\frac{1}{27} = 3^{-3}$

$$(3^2)^{x-2} = (3^{-3})^{2x+1}$$
$$2(x-2) = -3(2x+1)$$
$$2x-4 = -6x-3$$
$$8x = 1$$
$$\boxed{x = \frac{1}{8}}$$

Example 2. Solve:

$$4(8)^{4x} = \left(\frac{1}{32}\right)^{-x}$$

$4 = 2^2$

$8 = 2^3$

$\frac{1}{32} = 2^{-5}$

$2^2 \cdot (2^3)^{4x} = (2^{-5})^{-x}$

$2^2 \cdot 2^{12x} = 2^{5x}$

$2^{12x+2} = 2^{5x}$

$12x + 2 = 5x$

$7x = -2$

$x = \frac{-2}{7}$

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Example 3. Solve by guessing and checking. Give your answer to at least one decimal place of accuracy.

systematic approach = guess and check

$$(1.08)^x = 2$$

$(1.5, 1.12)$

$(2, 1.17)$

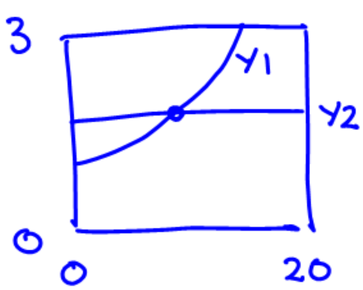
$(3, 1.26)$

$(9, 1.999)$

$x \doteq 9$

Solve this equation graphically

$\underbrace{1.08^x}_{y_1} = \underbrace{2}_{y_2}$

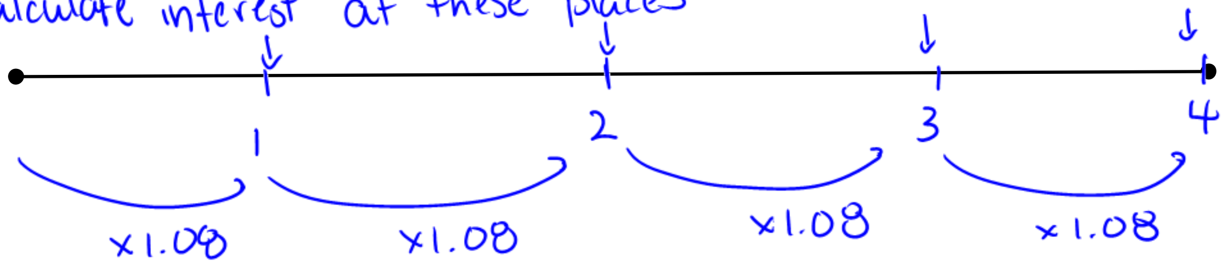


$(9.006, 2)$

$x \doteq 9.006$

Example 4. You have \$2000 invested at 8%/yr compounded annually. What will this investment amount to in 4 years?

calculate interest at these places



$y = 2000(1.08)^4$

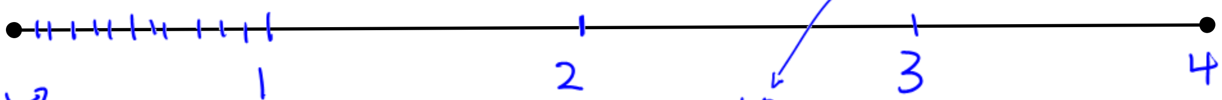
$= 2720.98$

$$Y = Y_0(a)^{\frac{x}{n}}$$

The formula for compound interest is $A = P(1+r)^n$, where A is the accumulated amount, P is the principal (amount earning interest), r is the interest rate per compound period, and n is the number of compound periods.

If in the previous question, the interest had been compounded monthly, what would the investment be worth in 4 years? each year is divided into 12 periods, so 48 periods in total. (4x12=48)

* only get $\frac{1}{12}$ of interest every month



$\times\left(1 + \frac{.08}{12}\right)$

$$y = 2000\left(1+\frac{.08}{12}\right)^{48}$$
$$= 2751.33$$

The compound interest formula is just a specific form of the general exponential function $y = y_0(a)^{\frac{x}{t}}$.

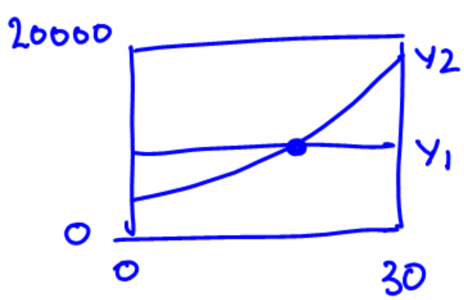
The formula $A = P(1+r)^t$ is sometimes called the annual compounding formula, and the general compound interest formula is then sometimes written as $A = P\left(1+\frac{r}{n}\right)^{nt}$ (where n represents the number of compound periods per year and t is the number of years.

Note: Be aware of the differences between what n, t and x represent in each of these formulas.

How long will it take \$5000 invested at 4.5%/a compounded quarterly to double in value?

$$Y = Y_0(a)^{\frac{x}{n}} \quad \text{or} \quad A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$\underbrace{10000}_{Y_1} = \underbrace{5000\left(1+\frac{.045}{4}\right)^{4x}}_{Y_2}$$



(15.49, 10000)

it will take 15.5 years.

p 364 #1-3, 7, 13, 14 quiz on □1-2 next day.