7.2 Transformations of Exponential Functions

Write the equation of the exponential function $y = 3^x$ after it has undergone each of the following transformations:

$f(x) = 3^x$

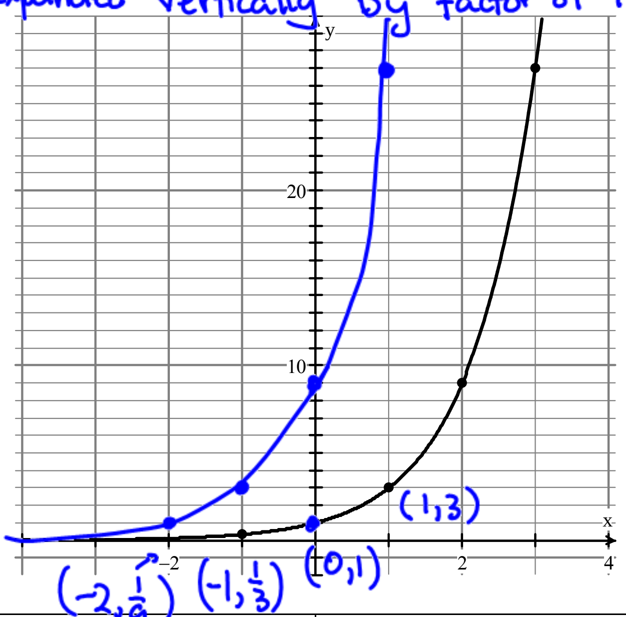
Transformation	***Equation***
Reflection in the *y*-axis $y = f(-x)$	$y = 3^{-x}$
Vertical expansion by 2, and a reflection in the *x-axis* $y = -2f(x)$	$\boxed{y = -2 \cdot 3^x}$
Translation 3 units up $y = f(x) + 3$	$y = 3^x + 3$
Translation 2 units right $y = f(x-2)$	$y = 3^{x-2}$

cannot do -6^x

Using the graph of $y = 3^x$, sketch the graph of each of the following. Give the domain, range, equation of the asymptote and the *y*-intercept of the transformed function.

$y = 9(3)^x$

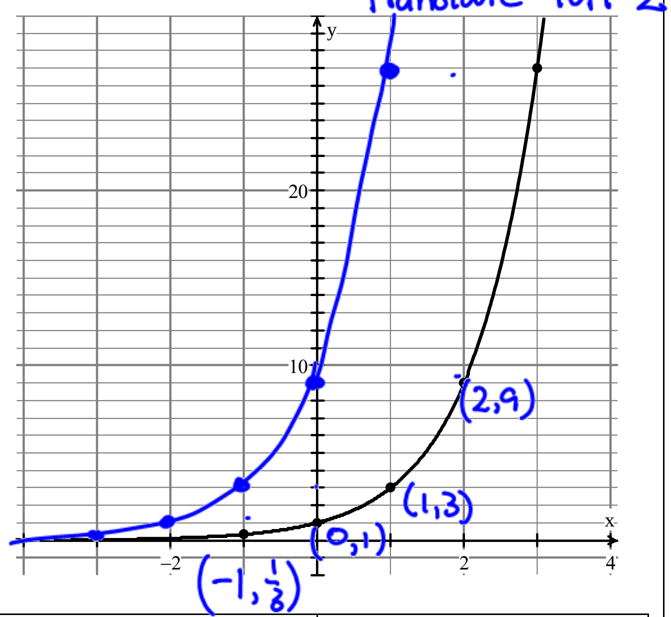
expanded vertically by factor of 9



Domain	Range
$x \in \mathbb{R}$	$y > 0$
***y*-intercept**	**Asymptote**
$(0, 9)$	$y = 0$

$y = (3)^{x+2}$

translate left 2



Domain	Range
$x \in \mathbb{R}$	$y > 0$
***y*-intercept**	**Asymptote**
$(0, 9)$	$y = 0$

$y = 9 \cdot 3^x$

$= 3^2 \cdot 3^x$

$y = 3^{x+2}$

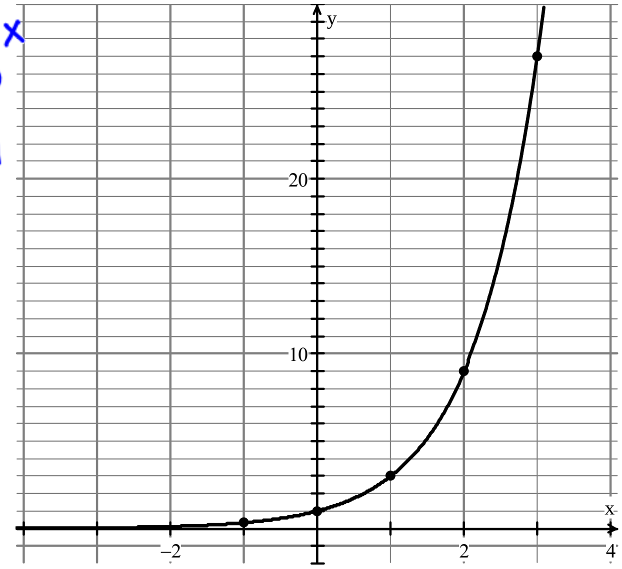
a vertical expansion will have the same graph as a translation (only for exponential functions)

$\frac{1}{3} = 3^{-1}$ $\frac{1}{9} = 3^{-2}$

$y = \frac{1}{3}(3)^x$

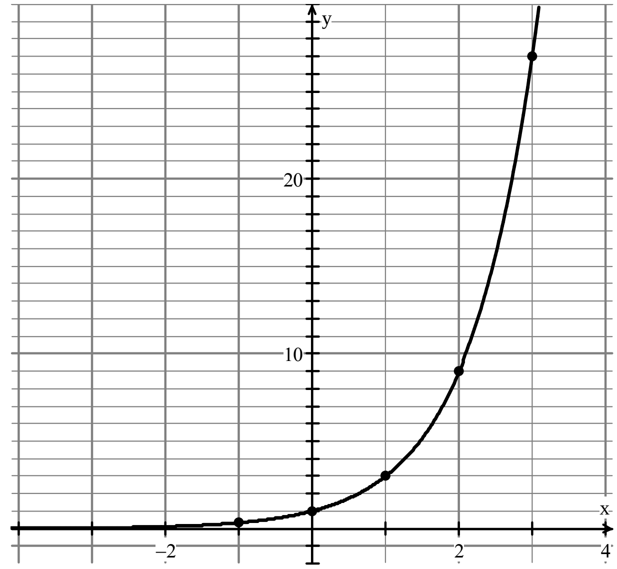
$y = 3^{-1} \cdot 3^x$

$y = 3^{x-1}$



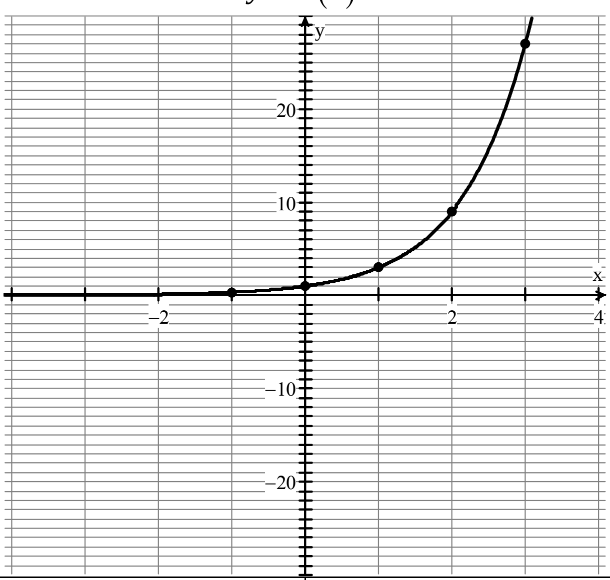
Domain	Range
y-intercept	Asymptote

$y = (3)^{x-1}$



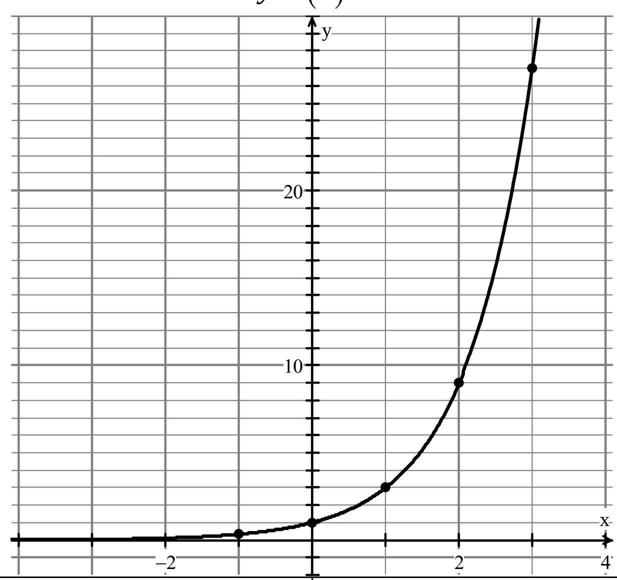
Domain	Range
y-intercept	Asymptote

$y = -(3)^x$



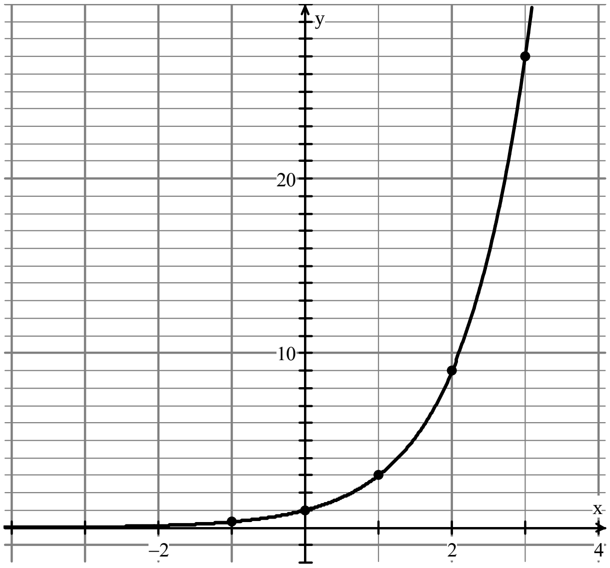
Domain	Range
y-intercept	Asymptote

$y = (3)^{-x}$



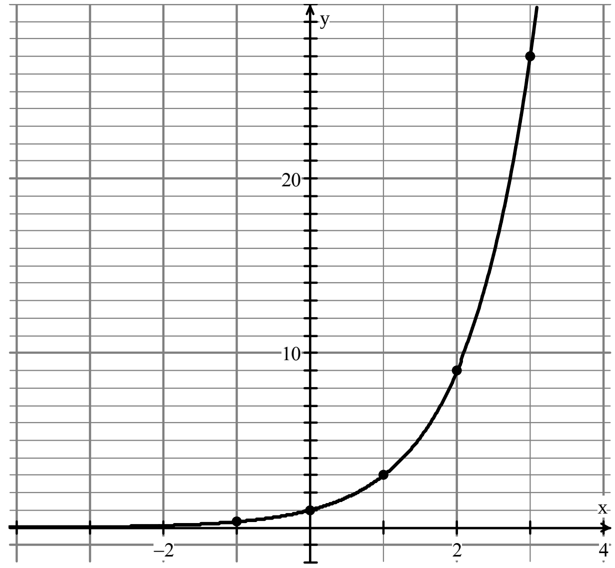
Domain	Range
y-intercept	Asymptote

$y=(3)^{2x}$



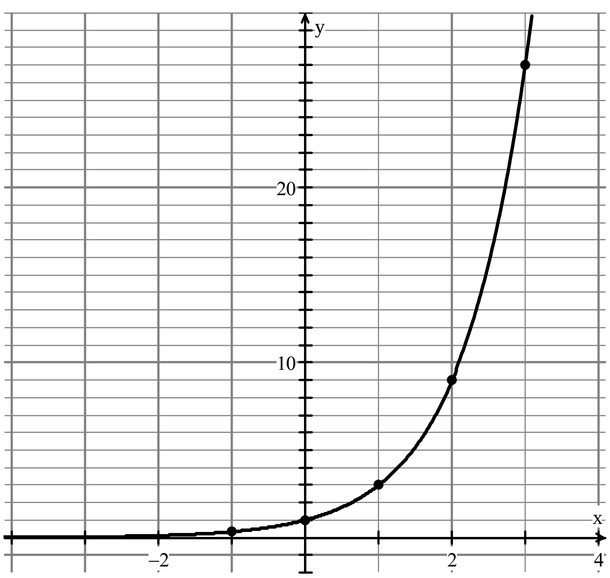
Domain	Range
y-intercept	Asymptote

$y=(3)^{\frac{1}{2}x}$



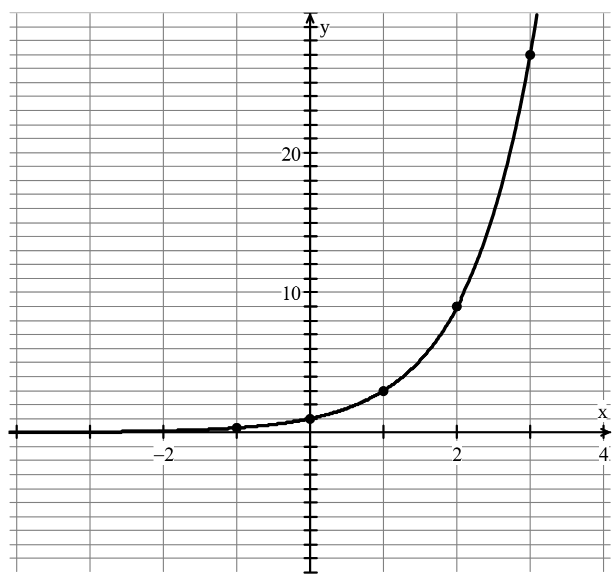
Domain	Range
y-intercept	Asymptote

$y=(3)^{x}+4$



Domain	Range
y-intercept	Asymptote

$y=0.5(3)^{-2(x+1)}-4$



Domain	Range
y-intercept	Asymptote

Some general observations

Stretching an exponential graph vertically can also be viewed as translating the graph horizontally.

Stretching an exponential graph horizontally can also be viewed as changing the base of the exponential function. This means that an exponential function can be rewritten with any positive base.

The transformed exponential function $y = y_0 (a)^{\frac{x}{t}}$ can be used to model situations where exponential growth or decay occurs. In this function, a represents the growth ($a > 1$) or decay ($0 < a < 1$) factor, y is the future (or past) amount, and y_0 is the initial or original amount (the amount at time 0). t is the amount of time it takes for 1 growth (or decay) period of factor a

Write an exponential function that could be used to represent each of the following

1. The population of Mathville doubles every 4 months. If the current population is 500, what will the population be in x months?

$$y = y_0 (a)^{x/t}$$

$$y = 500(2)^{\frac{x}{4}}$$

2. The population of a country is 8 million and growing at 2.13% per year. What will the population be in x years?

$$y = y_0 (a)^{\frac{x}{t}}$$

$$= 8 \text{ million} (1.0213)^{\frac{x}{1}}$$

3. Every 4 hours, your body removes 30% of a certain drug. If you have an initial dose of 120 mg, how many mg will remain in x hours?

$$y = y_0 a^{\frac{x}{t}}$$

$$y = 120(0.7)^{\frac{x}{4}}$$

* taking away 30% we retain 70%

4. A culture of bacteria doubles in size every 20 minutes. If the culture size is originally 8 cm^2, what size will the culture be in x minutes?

$$y = 8(2)^{\frac{x}{20}}$$

5. A car depreciates by 20% each year. If it is originally worth $30 000, what will its value be in x years?

$$y = 30000(0.8)^{\frac{x}{1}}$$